International Journal of Engineering, Science and Mathematics
Vol. 9 Issue 11, November 2020,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

FINDING CURRENT OF AN ELECTRICAL CIRCUIT USING GRAPH THEORETIC APPROACH AND NORTON'S THEOREM

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ABSTRACT

Graph theory is a branch of mathematics where we study about some relations between the objects. So it has many direct applications in real life problems. In Electrical circuit analysis also, different electrical elements are connected in a particular way. So both can be analysed with the help of each other. An electrical circuit can be represented by graph. From graph we can obtain different matrices. These matrices satisfy some matrix equilibrium equations of electrical circuit analysis. From these equations we can find many relatins between currents and voltages of electrical circuits. In our previous paper we analysed currents in a circuit with the help of Thevenin's theorem and graph theory. In this paper, branch currents of an electrical circuit with unknown resistances and voltage are obtained using network equilibrium equations and then it will be verified by Norton's theorem.

KEYWORDS:

Graph theory;
Electrical circuits;
Norton's theorem;
Branch current;
Loop current.

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1. INTRODUCTION:

Graph Theory is a branch of Mathematics that deals in some points and the connections between them. A graph is a set of ordered pair $G = (V, E)$ of sets where $E = \{\{x, y\} : x, y \in V\}$ is defined by a particular relation. The elements of V are called vertices (or nodes) of the graph G and the elements of E are called edges. So in a graph a vertex set is a set of points $\{x_1, x_2, x_3 \ldots .x_n\}$and edge is a line which connects two pointsx_i and x_j .

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In electrical engineering, an electrical circuit consists of some interconnected electrical devices. So circuit properties can be studied with the help of graph theory. In Electrical Engineering, Graph Theory is also used to verify the properties of different electrical circuits.Flowing current through the circuits obeys different laws such as Kirchhoff's current laws, Kirchhoff's voltage laws etc. Graph theory comes into account in the electrical circuits in the manner by which the elements or devices are connected in the circuits.

2. LITERATURE REVIEW:

A connected graph without closed path i.e. tree was implemented by Kirchhoff in 1847. He used graph theoretical concept in the calculation of currents in network or circuits. In 1892 this concept was improved by J.C.Maxwell [13]. The relation between Kirchhoff's methods for constructing the equations that describe an electric circuit and graphs was first clarified by Veblen in 1916. He showed Poincare's formal description of graphs by means of the incidence matrix [21]. In recent years electronic simulation programs determine the circuit equations by means of nodal analysis which shows a more direct way the relation between graphs and electric circuits.In 1980, Vandewalle [22] studied the application of coloured branch theorem in circuit theory. In the same year Satoru Fujishige developed an algorithm to solve graph realization problem. He used the fundamental circuit matrix or graphic matrix for developing it [15]. Istvan Fary in 1983 studied and developed some algorithms on generalised circuits [13]. In 1985, Karl Gustafson studied graph and networks using vector calculus and developed some results relating to divergence and curl [17]. In 1992, Brooks, Smith, Stone and Tutte gave an expression for current and potential differences using determinant [6]. In 1997, Gunther and Hoscheck adapted ROW method for simulation of electric circuits [16]. In 2012, Li and Xuan [19] used improved adjacency matrix for calculation of distribution network flow. In the same year bond graph was used to solve an electrical model by obtaining the system equations [20]. Harper [18] studied morphisms for resistive electrical networks to solve Kirchhoff's problem in 2014. Alman, Lian and Tran [1] in 2015 found a new result on circular planner electrical network.

3. FROM CIRCUIT TO GRAPH:

A graph can be obtained from an electrical circuit. We identify the graph G= (V, E) where V is the set of vertices and E is the set of edges. The edge between i^{th} and j^{th} vertices can be denoted by {i,j} ignoring the direction. Similarly the notation (i,j) can be used for oriented edges, where i is the start vertex and j is the end vertex. In general current source and voltage source are replaced by open circuit and short circuit respectively.

3.1 Matrices Associated to a Graph:

a) Fundamental Tie set matrix (Fundamental Loop Matrix) [B] :

This matrix is associated to a fundamental loop i.e. a loop formed by only one link (branch that does not belong to a particular tree) associated with other twigs (branch of trees). Here, all loops obtained from a particular tree forms the rows and all branches form the columns such that

$$b_{ij} = \begin{cases} 1, \; if\ branch\, b_j\ in\ loop\, i\ are\ in\ the\ same\ direction \\ -1, \; if\ branch\, b_j\ in\ loop\, i\ are\ in\ the\ opposite\ \ direction \\ 0, \; if\ branch\ \ b_j\ is\ \ not\ \ in\ \ loop\, i \end{cases}$$

b) Branch Impedance Matrix [Z_b]:

It is a square matrix of order m where m is the no of branches having branch impedance as the diagonal elements and mutual impedance as off diagonal elements. If there is no transformer or mutual sharing then off diagonal entries are zero.

3.2 Application of Graph theory in Network Equilibrium Equations:

If V_{SK} be the voltage source in a branch k having impedance z_k and carrying current i_k, then the branch voltage $v_k = z_k i_k + V_{SK}$

Tthis equation for the whole circuit can be written as $[V_b]=[Z_b][I_b]+[V_S]$ where $[Z_b]$ is the branch impedance matrix, $[I_b]$ is the column vector of branch currents and $[V_S]$ is the column vector of source voltage. Kirchhoff's Voltage law states that the algebraic sum of voltages in any closed path of network traversed in a single direction is zero. In matrix form it is represented as

$$[B][V_b]=0$$
$$\Rightarrow [B]([Z_b][I_b]+[V_S])=0$$
$$\Rightarrow [B][Z_b][I_b]=-[B][V_S]$$

Also the branch current matrix equation is $[I_b]=[B^T][I_L]$, where $[I_L]$ is the loop current matrix and $[B^T]$ is the transpose of [B]. Using this in above equation we have

$$[B][Z_b][B^T][I_L]=-[B][V_S]$$

We will use this equation to analyze the following circuits and also in the equivalent circuits obtained by Norton's theorem.

4. FINDING CURRENT USING GRAPH THEORETIC APPROACH AND ELECTRICAL CIRCUIT ANALYSIS:

Let us consider the circuit a circuit with one unknown resistance and one unknown voltage as follows

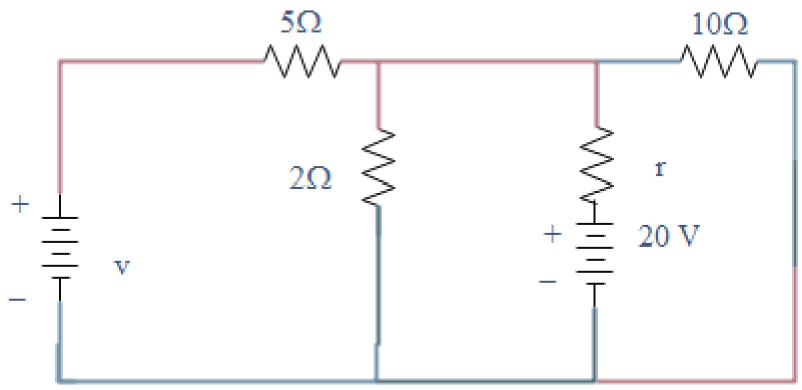

Figure 1: Circuit with one unknown resistance and one unknown voltage

$$r_1=5\Omega \qquad v_1=v\ \ V(Volt)$$
$$r_2=2\ \Omega \qquad v_2=20\ \ V$$
$$r_3=r\,\Omega$$
$$r_4=10\ \Omega$$

We will find the current through 10Ω resistor.

4.1 Graph theoretic approach:

The graph of the main circuit is

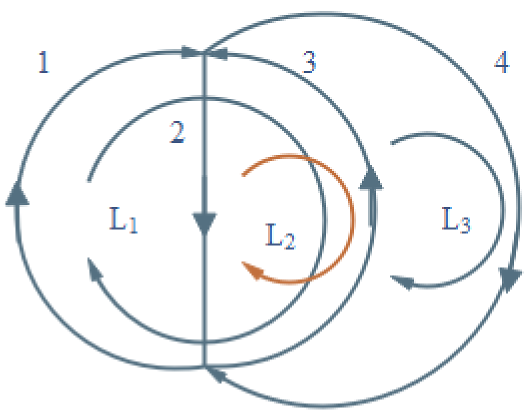

Figure 2: Graph of the circuit

Here considering branch 3 as tree branch we have

$$B=\begin{bmatrix}1 & 0 & -1 & 0\\ 0 & -1 & -1 & 0\\ 0 & 0 & 1 & 1\end{bmatrix},\quad Z_b=\begin{bmatrix}5 & 0 & 0 & 0\\ 0 & 2 & 0 & 0\\ 0 & 0 & r & 0\\ 0 & 0 & 0 & 10\end{bmatrix},$$

$$B^T=\begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ -1 & -1 & 1\\ 0 & 0 & 1\end{bmatrix},\quad I_L=\begin{bmatrix}I_{L_1}\\ I_{L_2}\\ I_{L_3}\end{bmatrix},\quad V_s=\begin{bmatrix}v\\ 0\\ 20\\ 0\end{bmatrix}$$

So putting in

$$[B][Z_b][B^T][I_L]=-[B][V_s]$$

$$\Rightarrow\begin{bmatrix}r+5 & r & -r\\ r & r+2 & -r\\ -r & -r & r+10\end{bmatrix}\begin{bmatrix}I_{L_1}\\ I_{L_2}\\ I_{L_3}\end{bmatrix}=\begin{bmatrix}20-v\\ 20\\ -20\end{bmatrix}$$

$$\Rightarrow X\ I_L=Y \quad where\ X=\begin{bmatrix}r+5 & r & -r\\ r & r+2 & -r\\ -r & -r & r+10\end{bmatrix},\ Y=\begin{bmatrix}20-v\\ 20\\ -20\end{bmatrix}$$

Solving we have

$$I_{L_1}=\frac{-0.6rv-v+20}{4r+5}A,\ I_{L_2}=\frac{0.5rv+50}{4r+5}A, I_{L_3}=\frac{-0.1rv-10}{4r+5}A,\ where\ 4r+5\neq 0$$

Again from

$[I_b]=[B^T][I_L]$ we have the branch currents as

$$\Rightarrow\begin{bmatrix}i_1\\ i_2\\ i_3\\ i_4\end{bmatrix}=\begin{bmatrix}\frac{-0.6rv-v+20}{4r+5}\\ \frac{-0.5rv-50}{4r+5}\\ \frac{v-80}{4r+5}\\ \frac{-0.1rv-10}{4r+5}\end{bmatrix}$$

So we see that $i_{10\Omega}=i_4=\dfrac{-0.1rv-10}{4r+5}A$

4.2 Verification by Norton's Theorem:

To verify the above result we find the Norton's equivalent circuit of the given circuit. We first consider 'v' as Source

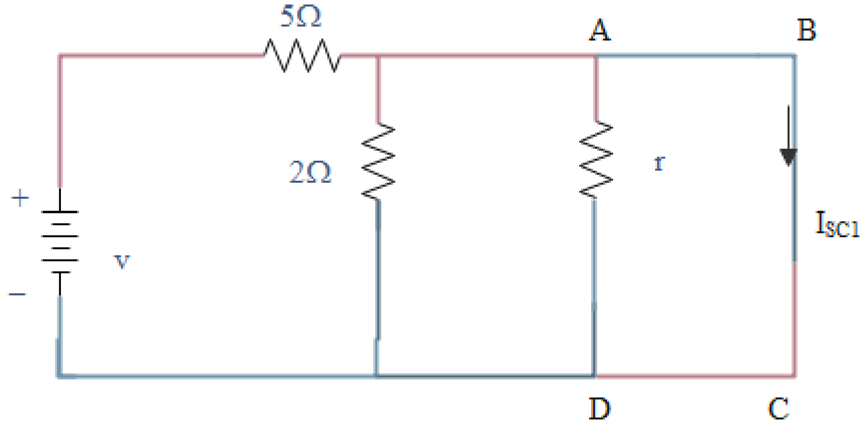

Figure 3: Circuit with voltage 'v' only

Here resistance at A-B-C-D = 0, as at point B resistance is zero due to short circuit and node A is parallel to node B.

So resistance (let R) in the loop $R = \frac{r.0}{r+0} = 0, r \neq 0$

So the circuit reduces to

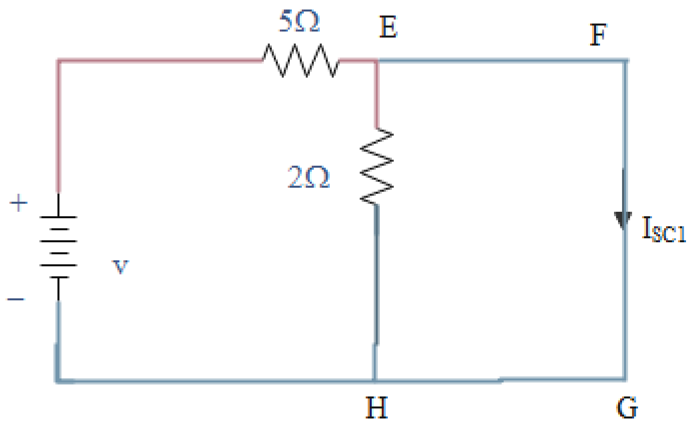

Figure 4: Reduced circuit

Using the same logic at node E and F the circuit further reduces to

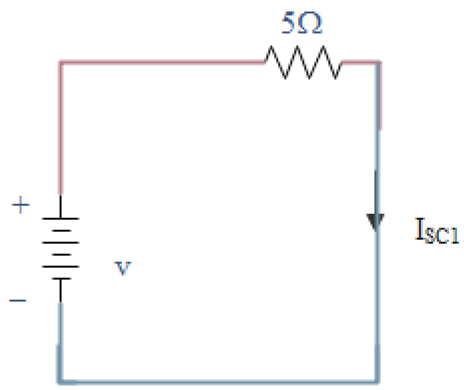

Figure 5: Reduced circuit

So $I_{SC1} = \frac{v}{5} A$

Now considering 20V as source and short circuiting 'v' source

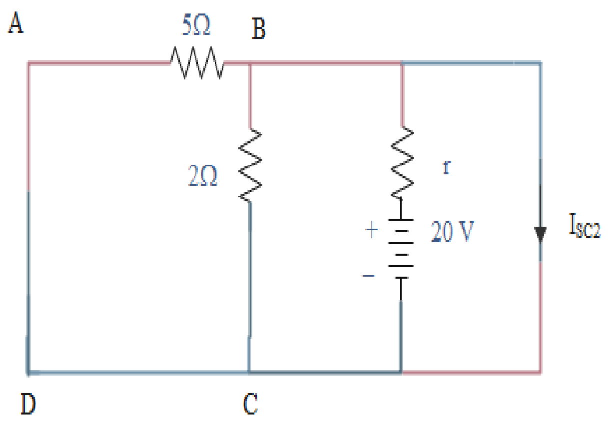

Figure 6: Circuit with voltage 20V only

Here in the loop A-B-C-D, total resistance $R = \frac{5 \times 2}{5+2} = \frac{10}{7}\ \Omega$

Using this circuit reduces to

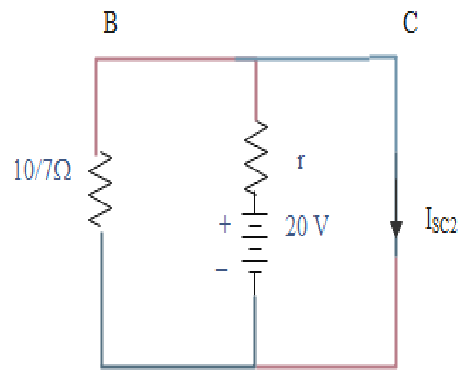

Figure 7: Reduced Circuit

The circuit can be redrawn as

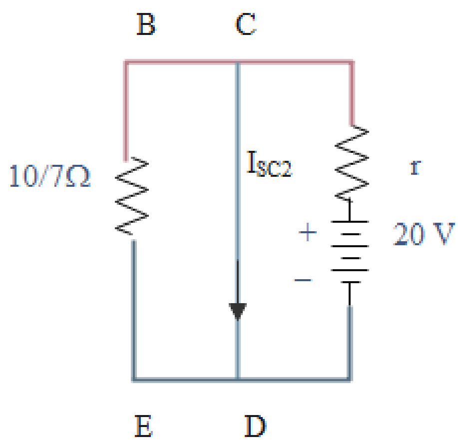

Figure 8: Reduced Circuit

Here resistance at B-C-D-E = 0, as at point C resistance is zero due to short circuit and node C is parallel to node B.
So the circuit further reduces to

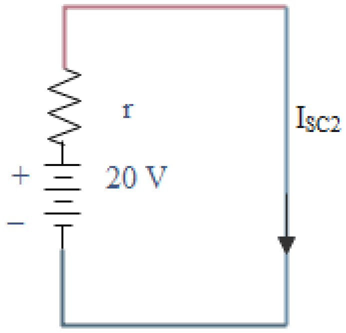

Figure 9: Reduced Circuit

So $I_{SC2} = \frac{20}{r} A$

So Norton's Equivalent Current, $I_N = I_{SC1} + I_{SC2} = \frac{v}{5} + \frac{20}{r} = \frac{vr + 100}{5r} A$

Now to find the internal resistance R_{int} of the circuit neglecting the 10Ω resistance we have

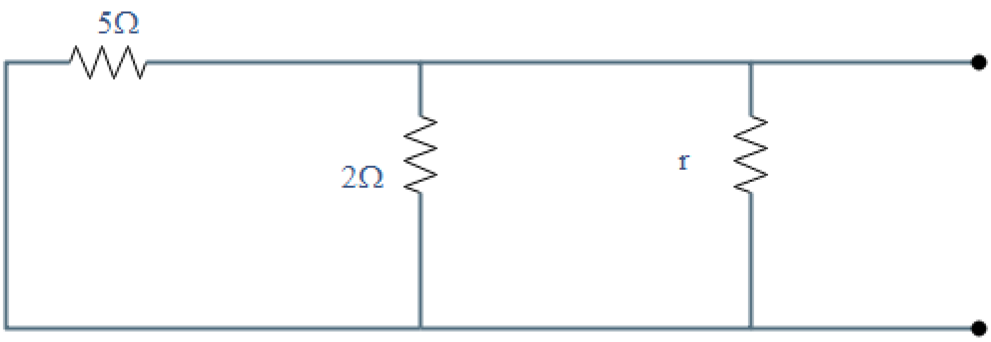

Figure 10: To find internal resistance

Since 5Ω, 2Ω, rΩ are in parallel so

$$\frac{1}{R_{\text{int}}} = \frac{1}{5} + \frac{1}{2} + \frac{1}{r}$$

$$\Rightarrow R_{\text{int}} = \frac{10r}{7r + 10} \Omega$$

So Norton's equivalent circuit is

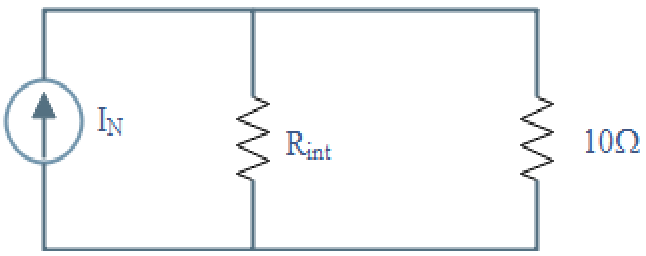

Figure 11: Norton's equivalent circuit

So by current division formula, current through 10Ω resistor (i_{10})

$$i_{10} = I_N \frac{R_{int}}{R_{int}+10} = \frac{rv+100}{5r} \cdot \frac{\frac{10r}{7r+10}}{\frac{10r}{7r+10}+10} = \frac{rv+100}{40r+50} A = \frac{0.1rv+10}{4r+5} A$$

In case of Matrix method negative sign is just showing that current is in opposite direction.
So here also we get the same branch current.

Results: In $X\, I_L = Y$ we see that

1. The matrix X is symmetric
2. The diagonal elements of X

   $a_{11} = r+5$ = sum of resistances in L_1

   $a_{22} = r+2$ = sum of resistances in L_2

   $a_{33} = r+10$ = sum of resistances in L_3

So the a_{ii} element (diagonal) of matrix X is the sum of the resistances of loop L_i of I_L.

3. Without Using nodal analysis and mesh analysis we can find the loop current and branch current by using graph theory only.

5. CONCLUSION:

From the above study we observed that the result obtained by Norton's Theorem is same as the result obtained by Graph theoretic approach in a circuit with unknown resistances and unknown voltage. So the graph theoretic approach is verified by existing Norton's theorem. The value of currents can be obtained for different value of resistances and voltages by network equilibrium equation. So with the help of graph theory we can easily find currents and voltages of electrical circuits. Graph theoretic approach is very beneficial when an electrical circuit contains many branches. It is laborious to use Norton's theorem to find branch current and loop currents etc in such cases. This difficulty can be solved with the help of matrix method because in this method we can multiply n^{th} order matrices which can be obtained from circuits containing n loops.

REFERENCE AND BIBLIOGRAPHY:

[1] Alman Joshua, Lian Carl, TranBrandon, Circular planar electrical networks: Posets and positivity, Journal of Combinatorial Theory, Series A 132 (2015) 58–101.

[2] Banerjee Anirban, Jost Jurgen (2008). On the spectrum of the normalized graph Laplacian, Linear Algebra and its Applications, Elsevier, 428 (2008) 3015–3022.

[3] Berdewar O. K., Dr. Deo S.D. Application of Graph Theory in Electrical Network, International Journal of Science and Research, Volume 5, Issue 3, March , 2016.

[4] Boruah Chanchal, Gogoi Krishna, Chutia Chandra. Analysis of Some Electrical Circuits with the Help of Graph Theory Using Network Equilibrium Equations, International Journal of Innovative Research in Science,Engineering and Technology,Vol. 6, Issue 1, January 2017.

[5]Boruah Chanchal, Chutia Chandra. Graph Theoretic analysis of an Electrical Circuit With Unknown Resistances And Voltages, International Journal of Engineering, Science and Mathematics(IJESM), Vol. 9, Issue 8, August, 2020, ISSN: 2320-0294.

[6] Brooks R.L., Smith C.A.B., Stone A.H., Tutte W.T., Determinants and current flows in electric networks, Discrete Mathematics 100 (1992) 291-301.

[7] Bryant Randal E. Symbolic Simulation—Techniques and Applications. Computer Science Department, Paper 240. (1990).

[8] Chakrabarti Abhijit. Circuit Theory Analysis and Synthesis, Dhanpat Rai & Co.

[9] Chang Ting-Jung, Tam Bit-Shun (2010). Graphs with maximal signless Laplacian spectral radius, Linear Algebra and its Applications 432 (2010) 1708–1733.

[10] Cvetkovic Dragos, Simic Slobodan K. (2010). Towards a spectral theory of graphs based on the signless Laplacian, II, Linear Algebra and its Applications,Elsevier, 432 (2010) 2257–2272.

[11] Deo Narsingh. Graph Theory with Applications to Engneering and Computer Science, Phi Learning Private Limited, New Delhi.

[12] Dorfler Florian, Simpson-Porko John W, Bullo Francesc.(2018).Electrical Networks and Algebraic Graph Theory: Models, Properties and Applications, Proceedings of the IEEE, Vol. 106, No.5, May 2018.

[13] FARY Istvan, GENERALIZED CIRCUITS. ELEMENTS OF MATHEMATICAL THEORY, Journnl of Pure and Applied Algebra 29 (1983) 241-283.
[14]Fayez Mohamed, Graph Theory and its application in Electrical Power System, ResearchGate, 2016.

[15] Fujishigi Satoru, An Efficient PQ-Graph Algorithm for Solving the Graph-Realization Problem, JOURNAL OF COMPUTER AND SYSTEM SCIENCES 21, 63-86 (1980).
[16] Gunther M., Hoschek M., ROW methods adapted to electric circuit simulation packages, Journal of Computational and Applied Mathematics 82(1997), 159-170.
[17] Gustafson Karl, Harary Frank, THE CURL OF GRAPHS AND NETWORKS, Mathematical Modelling, Vol. 6, pp 145-155, 1985.
[18] Harper L.H., Morphisms for resistive electrical networks, Discrete Applied Mathematics 163 (2014) 181–193, Elsevier.
[19] Li LIU, Xuan ZHAO, Application of Improved Adjacency Matrix Multiplication in Distribution Network Flow Calculation, Energy Procedia 14(2012) 983-989.
[20] Šarga Patrik, Hroncová Darina, curilla Miloslav, Gmiterko Alexander, Simulation of electrical system using Bond Graphs and MATLAB/Simulink, Procedia Engineering 48 (2012) 656 – 664, Elsevier.
[21] Toscano L, Stella S, Milotti E, Using graph theory for automated electric circuit solving, available online.
[22] Vandewalle Joos, The Coloured Branch Theorem and Its Applications in Circuit Theory, IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS . VOL. CAS- 27, NO 9, SEPTEMBER 1980.
[23] Wang Wei, Xu Cheng-Xian (2010). On the asymptotic behavior of graphs determined by their generalized spectra, Discrete Mathematics 310 (2010) 70-76.
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